Gravitational Deflection of a Light Beam in the Reciprocal System

Prof. K.V.K. Nehru, Ph.D.

The gravitational deflection of light beam owes its origin to the same factor as that causing the excess perihelion precession of the planets—namely, the coordinate time component associated with independent motion.¹ But there is a significant difference between the movement of a planet and the movement of a photon in the sun's gravitational field. In the former case, the motion of the planet is an independent motion. On the other hand, the motion of the photon is due to the background space-time progression and is introduced by our use of the stationary reference system. This has an important bearing on the manner in which the *spatial effect* of the coordinate time manifests itself in the motion, in the two cases, as will be explained below.

1 Gravitational Motion and the Gravitational Potential

The gravitational motion of a material atom is inward in space. But in a celestial object like a star, which is an spatial aggregate of such units, the inward motion of each unit is counterbalanced by the interaction with the contiguous neighbors. The scalar space-time direction of this counterbalancing force is in opposition to that of gravity and has the same magnitude as the gravitational motion and is equal to the escape velocity, v, at that location. The escape velocity can be evaluated by noting that the centrifugal force on a mass m situated at a radial distance r will be equal to the gravitational force on it by the central mass M. Thus

$$\frac{m v^2}{r} = G \frac{M m}{r^2} \quad or, \quad v^2 = \frac{G M}{r} \tag{1}$$

where G is the gravitational constant.

2 Coordinate Time

The coordinate time increase associated with a speed v is given by

$$\frac{v^2}{c^2} \frac{fraction of unit}{unit}$$
 (2)

This is in the radial direction of the counterbalancing force explained in the paragraph above.²

Let the radial distance of the photon at its closest approach to the sun be r_0 . Since v^2 is a point function of the radial distance given by Equation (1), the increase in the coordinate time for a change of radial distance from the "outer gravitational limit" to r_0 will be given by

¹ Larson, Dewey B., Nothing But Motion, North Pacific Pub., Portland, Oregon, 1979, p. 99-100.

² Larson, Dewey B., Beyond Newton, North Pacific Pub., Portland, Oregon, 1964, p. 126.

³ Larson, Dewey B., Quasars and Pulsars, North Pacific Pub., Portland, Oregon, 1971, p. 166.

$$\tau_{g-0} = \left(\frac{v_0^2}{c^2} - 0\right) = \frac{v_0^2}{c^2} \tag{3}$$

where $v_0^2 = GM/r_0$. The circumferential space equivalent² of coordinate time increase is $\pi v_0/c^2$. But the photon is already moving at unit speed—one unit of space per unit of time—in the forward dimension. As such no further spatial shift is possible in the direction of its motion (unlike in the case of the planetary motion). However, in view of its scalar nature, the spatial effect of this coordinate time increase will manifest in a spatial dimension other than the one in which the photon is already progressing at unit speed. Thus the photon gets displaced in the inward radial direction coinciding with the direction of gravity.

Now the question arises why this effect should manifest radially inward instead of radially outward direction. The situation here can be easily understood if we look to an analogy from the motoring/generation principle in electrical engineering. Current flowing in a particular direction, in the conductor of a motor armature situated in a magnetic field forces the armature to rotate. But the rotation of the conductor (in the same magnetic field) now generates what we call the "back e.m.f." and causes a current flow in the conductor in the opposite direction (opposing the original current), establishing a natural equilibrium. Analogously, in the present case, the coordinate time increase resulting from a radially outward equilibrium motion manifests as a circumferential spatial shift. While this gives rise to the excess perihelion motion in the case of orbiting planets, in the case of the photon such motion not being possible, the spatial shift, $\pi v_0/c^2$, shows up in the radial direction opposite to that of the originating motions that is, it manifests in the radially inward direction.

We have so far considered the increase in coordinate time only during half of the transit, from the outer gravitational limit to r_0 . The coordinate time change associated with the remaining journey, from r_0 onwards to the outer gravitational limit on the other side, will similarly be

$$\tau_{0-g} = \left(0 - \frac{v_0^2}{c^2}\right) = \frac{-v_0^2}{c^2} \tag{4}$$

This will again manifest as a spatial shift of magnitude $\pi v_0/c^2$. It must be noted that the negative sign of the coordinate time increase, in Equation (4) above, has no relevance in deciding the direction of its spatial effect. The spatial effect is always additive, irrespective of the sign of the coordinate time because of the scalar nature of the relation between the dimensions of time and the dimensions of space. Thus the total spatial shift in the direction perpendicular to that of progression is given by

$$\delta_0 = 2\pi \frac{v_0^2}{c^2} \tag{5}$$

in fraction of unit/unit or simply the deflection in radians.

3 Interaction Cross-section

However, this deflection, given by Equation (5) is not necessarily effective in its entirety. This requires the consideration of the way in which an independent motion, as against the fictitious motion of the space-time progression, can be brought to bear on a photon or a material particle. An independent motion can be imparted to a material atom, for example, because it can offer a *resistance* in the

direction of the motion being applied. The resistance to motion is due to the speed displacement in that dimension. For instance, why we don't find sub-atomic particles participating in the scalar inversion from the cosmic sector to the material sector, giving us the cosmic rays is because they are unable to build up speed in the vacant dimension in which they do not have any displacement. In contradistinction, the motion of the space-time progression applies in the vacant dimension, as in the case of a photon, for example.

As such, in the present case, the full force of the deflection motion is applicable to the photon only if the plane of vibration of the photon is parallel to the deflection motion: that is, if it is in the direction of the gravity.

Let us take a look at the photon in the direction of its progression. Referring to the figure, let the direction perpendicular to the plane of the paper represent the direction of the photon progression. The diameter of the circle is one natural unit of space representing the amplitude of the photon vibration. Any diameter of the circle, like PP, now represents the plane of vibration of the photon (looking end-on). OD is the direction of the deflection motion. Suppose the photon vibration happens to be in the YD direction, the full impact of the deflection motion, δ_0 , can be imparted to it. On the other hand, if the plane of vibration is XB, since the photon does not carry any displacement in the YD direction, none of the deflection motion can be imparted to the photon. In fact, when the plane is tilted at an angle φ to YD, the fraction of the deflection motion that can be transferred to the photon is proportional to $\cos \varphi$.

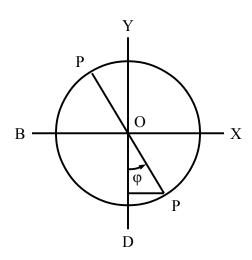


Figure 1

In an unpolarized beam all orientations are equally existent and the average value of the resistance—which I will call the "interaction cross-section"—that makes the motion transfer possible can be obtained by

$$\sigma = \frac{\int_{-\pi/2}^{\pi/2} \cos(\phi) d\phi}{(\pi/2 - (-\pi/2))} = \frac{2}{\pi}$$
 (6)

So finally, from Equations (5) and (6), the total effective deflection is

$$\sigma = \left(2\pi \frac{v_0^2}{c^2}\right) \left(\frac{2}{\pi}\right) = 4\frac{v_0^2}{c^2} \tag{7}$$

Or, using Equation (1), we have

$$\sigma = 4 \frac{GM}{R_0 c^2} \tag{8}$$

4 Polarized Beam

It may be noted that the above result is identical to what General Relativity predicts. However, the result differs from the Relativity value in the case of polarized beam of radiation. Consider the case of a fully polarized beam. Let the plane of polarization be represented by PP (Figure 1), inclined at angle ϕ to the direction of gravity, YD. From what has been said above, the total effective gravitational deflection will be

$$\sigma_{p} = \frac{2\pi v_{0}^{2}}{c^{2}} \cos(\phi) = 2\pi \frac{GM}{r_{0}c^{2}} \cos(\phi)$$
(9)

In the more general case where the degree of polarization in each direction varies, we proceed as follows. Let the power p in any plane (of polarization) be a function of the tilt angle φ :

$$p = p(\varphi)$$
.

Then the average interaction cross-section is given by

$$\sigma_p = \frac{\int_{-\pi/2}^{\pi/2} p(\phi) \cos(\phi) d\phi}{\int_{-\pi/2}^{\pi/2} p(\phi) d\phi}$$
(10)

The total effective deflection, then, is

$$\delta_{\rm p} = \delta_{\rm o} \, \sigma_{\rm p}$$

This aspect of the theory, namely, the dependence of the gravitational deflection of the polarization characteristics of the traversing beam provides a possibility to observationally test it in comparison with the theory of Relativity.